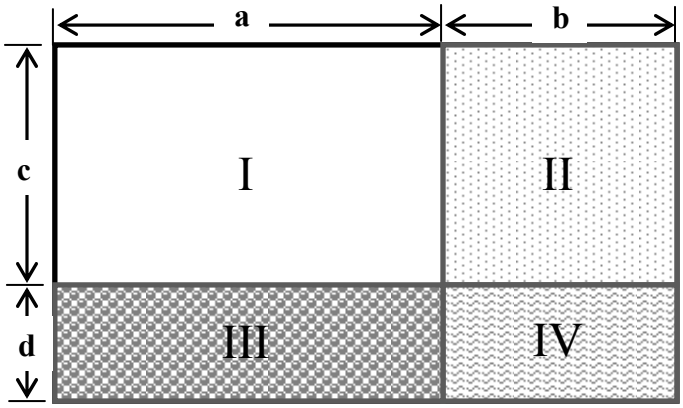
Investigation Distributing the Area



The large rectangle shown here is cut by lines parallel to its sides into four smaller rectangles. Use this area model to multiply and factor algebraic expressions. Each question below is based on this diagram.

The big idea is that the area of a large rectangle can be found two ways:

- Multiply the length (a+b) by the width (c+d).
- Find the area of each of the four smaller rectangles and add them.

Determining the area

1. Suppose a=8, b=3, c=5, and d=2.
   a. Draw a diagram and label it with these numbers.
   b. Determine the area of each smaller rectangle I, II, III, and IV.
   c. Determine the total area of the large rectangle.

2. Suppose area I = 28 square units, a = 7, d=2, and area IV= 6 square units.
   a. Draw a diagram and label it with the numbers.
   b. Determine the missing lengths b and c.
   c. Determine the areas of the smaller rectangles II and III.
   d. Determine the total area of the large rectangle.

3. Suppose a and c are the same length x.
   a. Which of the rectangles must be a square? Explain why.
   b. Write an expression for the area of the square.

4. Suppose area I is a square with an area of 81 square units. If b=4 and d=3, find the missing lengths and areas, including the total area.

5. Suppose area I is a square with side length x, area III= 8x square units, and area IV= 24 square units. Draw a diagram and label it with the given information.
   a. Determine the missing lengths.
   b. Write algebraic expressions for the length and width of the large rectangle.
   c. Determine the areas of the smaller rectangles
   d. Write two equivalent expressions for the area of the large rectangle.

6. Show how the area model can be used to multiply each of the following. Write two equivalent expressions for the area of the large rectangle.
   a. $(x+3)(x+7)$ b. $(x + 4)(x – 5)$ c. $(x+2)(2x+1)$ d. $(x – 3)(x – 5)$

Adapted from **Fireworks** by Fendel, et al., *Interactive Mathematics Program: Integrated High School Mathematics, Year 2.* Emeryville, CA: Key Curriculum Press, 2010. Print. Pg.19

Determining the dimensions

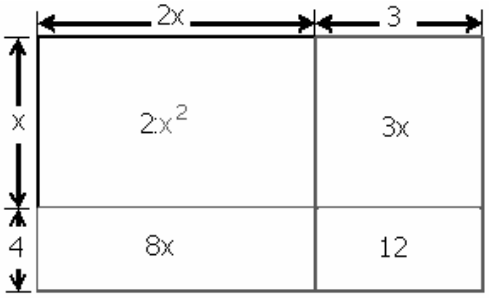
7. Suppose the total area is represented by the expression $x^2 + 5x + 3x + 15$,
   a. Draw a diagram and label it according to the information provided in the expression.
   b. Determine the missing lengths.
   c. Write an algebraic expression for the total area of the large rectangle as a product of the length and width.

8. Suppose the total area is represented by the expression $x^2 + 14x + 24$. If area I is x^2 square units and area IV is 24 square units,
   a. what must be true about area II and area III?
   b. what must be true about lengths b and d?
   c. Write algebraic expressions for the length and width of the large rectangle.
   d. Write an algebraic expression for the total area of the large rectangle as the product of the length and width.

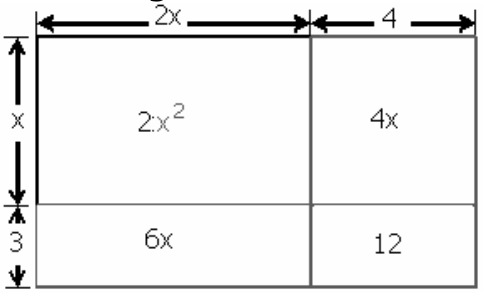
9. Consider each of the following expressions as representing the total area of the large rectangle. Write an equivalent expression for the total area as a product of the length and width.
   a. $x^2 + 7x + 12$ b. $x^2 + 10x + 21$
   c. $x^2 - 9x + 20$ d. $x^2 + 5x - 24$

10. Quadratic expressions are commonly written in the form ax^2+bx+c where $a \neq 0$. Students were using the area model to write equivalent expressions for $2x^2+11x+12$. Liam's and Margo's models are given below.

Liam's Solution



Margo's Solution



   a. How are the models similar and how are they different?
   b. Which model is correct?
   c. Using the model you determined was correct, write an equivalent expression for $2x^2+11x+12$ as the product of two linear factors.

Summarize the Mathematics

In this investigation you used an area model to write equivalent quadratic expressions.

   a) Draw an area model for $(x + m)(x + n)$.
   b) Write two equivalent expressions for the total area.
   c) Describe how the values of m and n can be determined if the total area of the large rectangle is given as $x^2 + bx + c$.

Adapted from **Fireworks** by Fendel, et al., *Interactive Mathematics Program: Integrated High School Mathematics, Year 2.* Emeryville, CA: Key Curriculum Press, 2010. Print. Pg.19

Check Your Understanding

a) Draw an area model for each of the following expressions and write two equivalent expressions for the total area.
   i. $(x+3)(x+2)$
   ii. $(m+2)(m-4)$
   iii. $(5 + r)(-2 + r)$
   iv. $(n+4)(n+5)$

b) Find values for the missing numbers that will make the given expressions equivalent.
   i. $x^2 + 12x + \_\_\_\_ = (x +4)(x + \_\_\_)$
   ii. $x^2 + \_\_\_x - 8 = (x + 4)(x + \_\_\_)$

c) Determine the length and width of the rectangle whose area is represented by the expression $x^2 + 8x + 7$.

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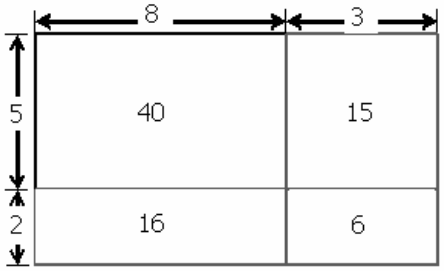
Investigation Distributing the Area (Teacher Notes)

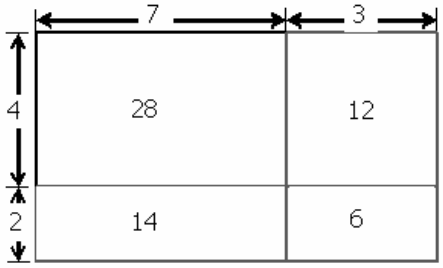
NC CCSS Math 1: Seeing structure in expressions
A-SSE.1 Interpret expressions that represent a quantity in terms of its context.*
A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
NC CCSS Math 1: Arithmetic with Polynomials & Rational Expressions
A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
NC CCSS Math 1: Interpreting Functions
F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function

1.



Area = 40 + 15 + 16 + 6 = 77

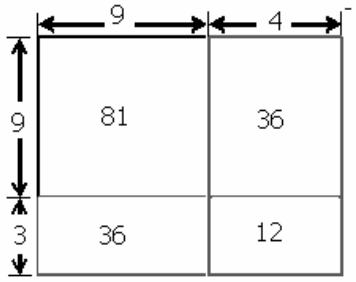
2.



Area = 28 + 12 + 14 + 6 = 60

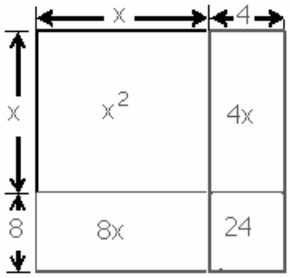
3. Area I is a square since a and c make up the sides. The area of the square is x^2.

4.



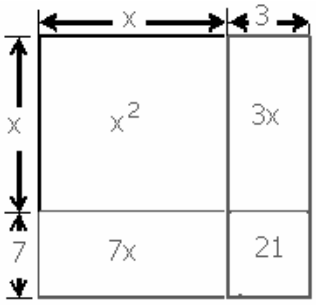
Area = 81 + 36 + 36 + 12 = 156

5.



Area = (x + 4)(x + 8)
Area = $x^2 + 4x + 8x + 24$
$= x^2 + 12x + 24$

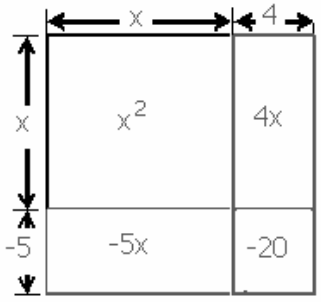
6a.



Area = $x^2 + 3x + 7x + 21$
$= x^2 + 10x + 21$

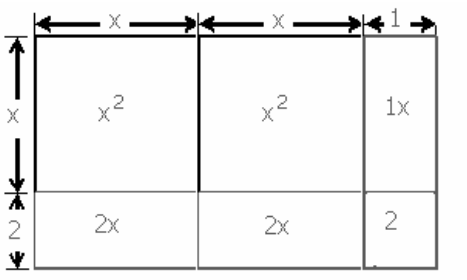
Adapted from **Fireworks** by Fendel, et al., *Interactive Mathematics Program: Integrated High School Mathematics, Year 2.* Emeryville, CA: Key Curriculum Press, 2010. Print. Pg.19

6b. Students may be creative in dealing with negative areas. Most students are comfortable with a diagram like this even though negative lengths and area are shown.



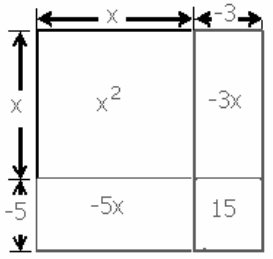
Area = $x^2 + 4x - 5x - 20$
= $x^2 - 1x - 20$

6c.



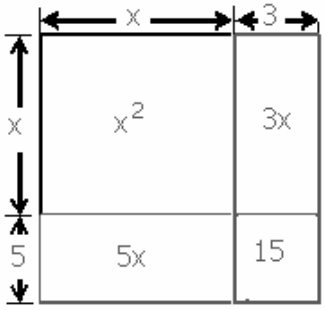
Area = $x^2 + x^2 + 1x + 2x + 2x + 2$
Area = $2x^2 + 5x + 2$

6d.



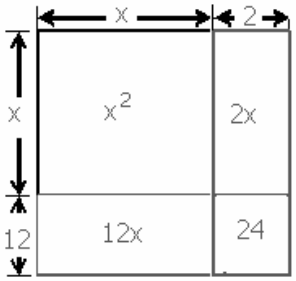
Area = $x^2 - 3x - 5x + 15$
Area = $x^2 - 8x + 15$

7.



Area = $(x + 3)(x + 5)$

8. a. Area II and Area III must add up to 14x square units.
b. b & d must multiply to give the area of 24 square units
c. (x+2) and (x + 12)
d. (x+2)(x+12)



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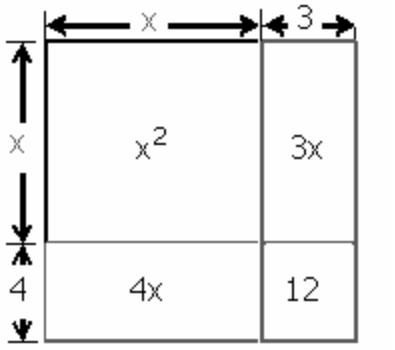
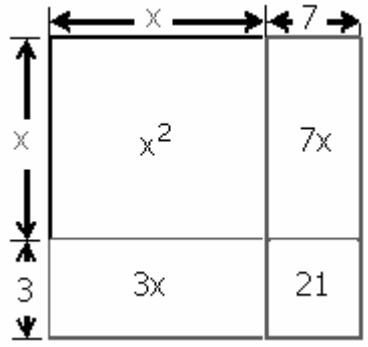
9a.



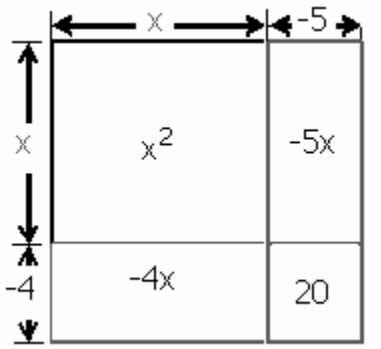
Area = (x+3)(x+4)

9b.



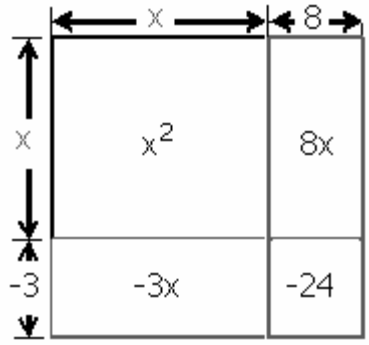
Area = (x+7)(x+3)

9c.



Area = (x – 5)(x – 4)

9d.

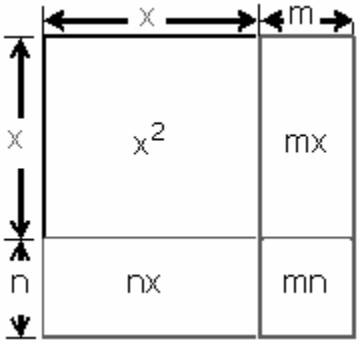


Area = (x + 8)(x – 3)

10. Both models have $2x^2$ for area I and 12 for area IV. The difference is where the 3 and 4 are arranged. This changes the areas of II and III. Liam's is correct because 8x + 3x is 11x whereas Margo has only 10x. The equivalent expression is (2x + 3)(x+4).

Summarize the Mathematics

a)



b) Area = $x^2 + mx + nx + mn$
$= x^2 + (m+n)x + mn$

c) m and n must add to be the value of b and m times n must be the value of c

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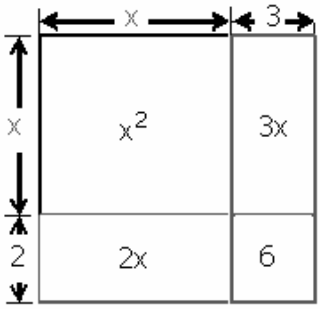
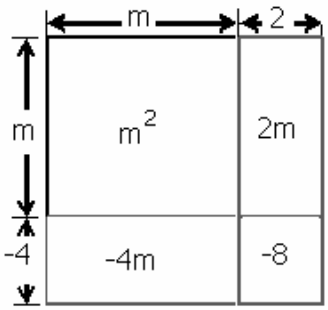
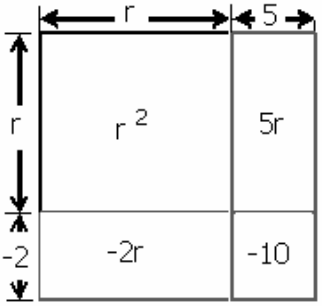
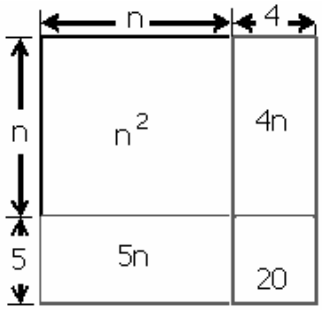
Check Your Understanding

a)









i) area = $x^2+3x+2x+6$
= x^2+5x+6

ii) area = $m^2+2m-4m-8$
= m^2-2m-8

iii)area = $r^2+5r-2r-10$
= $r^2+3r-10$

iv) area= $n^2+4n+5n+20$
= $n^2+9n+20$

b)

i) $x^2 + 12x + \underline{32} = (x +4)(x + \underline{8})$

ii) $x^2 + \underline{2}x - 8 = (x + 4)(x + \underline{-2})$

c) (x+1)(x+7)

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